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RESEARCH IDEAS FOR STUDENTS : TOPOLOGY OF THREE DIMENSIONAL FLOWS

- What are the possible boundaries for chaotic attractors?
In three manifold theory, from works of Thurston and Perelman, we know we can decompose any three manifold canonically, so that each piece has a definite geometry.

  In chaos people often draw attractors that are floating in $\mathbb{R}^n$ (the phase space for a chaotic system), but this does not allow for a good topological characterization of these systems. I suspect there's always a way to decompose the manifold into geometric pieces, so that in a chaotic piece there's a single attractor, and its boundary sits on the boundary of this piece.

  As a first step, before trying to prove a general theorem, I'd like consider a few examples. I have one example that I started to work on, the Lorenz system (see below) but there are many other important three dimensional flows we can try and analyse using the same tools.
- Relation to Anosov flows. For this example of the Lorenz system, I conjecture that the flow is topologically equivalent to a very different kind of flow: The geodesic flow on the modular surface. Two important ingredients here are:
- a work (in progress, joint with Adam Clay), classifying all possible Anosov flows on a particular three manifold (the trefoil complement), and
- a classification by Birman, Williams, Ghys and others of all possible periodic orbits for the geodesic flow on the modular surface.

  Even for the Lorenz system other three manifolds appear, but for these we have no understanding of the possible Anosov flows. So this leads to other questions, for example:
1. Can we classify all Anosov flows on other knot complements?
2. What kind of orbits appear for geodesic flows on different surfaces, or more general Anosov flows?

  Here's a paper explaining many of the ideas:
On the Topology of the Lorenz System,