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Max Gurevich - Representation Theory - for students

Representation theory sits at the crossroads of many mathematical subjects, such as algebra, Lie theory, number theory, harmonic analysis, mathematical physics (to name a few...), while maintaining its own flavor and showing intrinsic elegance and interest.

Often we would like to study a group (because we know that groups are the mathematical manifestation of symmetry) with a complicated structure/geometry. One could try understanding the group by looking at its actions. Representation theory comes into play when the actions turn out to be linear, that is, when we can use linear algebra to study the action.

On other occasions we would like to study a specific vector space, perhaps, a space of some curious functions. If that space, as often happens, has a natural action of a group, we can harness the power of the representation theory of that group to tackle our problem.

Even more, familiar groups which are easily defined may carry immense mathematical depth inside the structure of their representations. This structure asks to be studied for its own sake or for its applications through the links mentioned above. A classical example would be the finite group of permutations on n symbols. Certain aspects of the linear actions of this group remain unsolved mysteries, which fuel developments of new theories.

My own research is focused on the representation theory of p-adic groups. These may be viewed as "number-theoretic analogs" of Lie groups, for example, the group of invertible matrices over a field with a totally disconnected topology. The nature of these groups allows algebra and combinatorics to supplement the classical tools in Lie theory.

I am especially fond of observing the links between structures (or, categories) which arise in different settings. Somewhat philosophically, p-adic groups can also be thought of as quantum versions of more classical objects, such as the permutation groups from above. We can make this philosophy concrete by comparing their representation theory with phenomena in a subject called quantum groups, which rose naturally from questions in physics.

I would be happy to meet and advise students who are interested in pursuing research and studies of these themes.