HARMONIC ANALYSIS ON EUCLIDEAN SPACES AND SOME OF ITS APPLICATIONS

INSTRUCTOR : AMOS NEVO

Fall semester, 2020-2021

1. Topics of the course

(1) Basic theory of Fourier series on the circle: orthogonality, mean convergence, summability kernels, pointwise convergence
(2) Basic theory of Fourier transform on the real line : L^1-theory and L^2 theory, Plancherel's theorem and the inversion formula,
(3) A more general perspective : unitary characters and Fourier transform on more general Abelian groups : from finite fields to Euclidean lattices to higher dimensional Euclidean spaces,
(4) Fourier transform of the measure on the sphere, and the radial Euclidean convolution structure,
(5) Diverse applications of Fourier Analysis (as time permits)
   - Ergodic theory on higher dimensional tori : Kronecker's theorem, spherical equidistribution,
   - The Poisson summation formula and lattice point counting problems,
   - The Heisenberg uncertainty principle for the Fourier transform.

Prerequisites. We will assume familiarity with the basic results of the courses on real functions / measure theory, introduction to functional analysis, and metric topology.

Time : Sunday 11:30-13:30, Wednesday 11:30-12:30

Grade : Final exam of 3 hours at the end of the semester.

2. Some references

(1) E. M. Stein, Fourier Analysis - an introduction.
(2) Y. Katznelson, An Introduction to Harmonic Analysis.
(3) E. M. Stein, Harmonic Analysis on Euclidean Spaces.
(4) G. Folland, A Course in Abstract Harmonic Analysis.