ALGEBRAIC GEOMETRY FOR PROSPECTIVE STUDENTS

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At its heart, algebraic geometry studies the geometry of algebraic varieties: the zero sets of systems of polynomial equations with coordinates in a given field $\mathbb{K}$. If the system consists of linear polynomials, then we all know everything about the zero set: it's just another linear space. But if the degrees of our defining polynomials get larger, the geometry becomes more interesting and complicated so that just determining its dimension, number of pieces (i.e. irreducible components), and singularities can become both theoretically and computationally difficult. The theorems of algebraic geometry provide the tools for answer these questions in general or for particular "types" of polynomials using the tools of abstract algebra and category theory (at least in modern times).

To illustrate what makes the study of algebraic geometry so compelling, let us quote Shafarevich [1]:

"Algebraic geometry played a central role in 19th century math. The deepest results of Abel, Riemann, Weierstrass, and many of the most important works of Klein and Poincaré were part of this subject" . But by the 20th century, "algebraic geometry had become set in a way of thinking too far removed from the set-theoretic and axiomatic spirit that determined the development of math at the time. It was to take several decades, during which the theories of topological, differentiable and complex manifolds, of general fields, and of ideals in sufficiently general rings were developed, before it became possible to construct algebraic geometry on the basis of the principles of set-theoretic math. Towards the middle of the 20th century algebraic geometry had to a large extent been through such a reconstruction. Because of this, it could again claim the place it had once occupied in math. The domain of application of its ideas had grown tremendously, both in the direction of algebraic varieties over arbitrary fields and of more general complex manifolds.... The foundation for this reconstruction was algebra."

Algebraic geometry continues to find applications in combinatorics, representation theory, and mathematical physics. My research focus on themes and tools inspired by the mirror symmetry phenomenon in string theory. Although I don't study the physics itself, many of the recent great ideas in algebraic geometry such as Gromov-Witten theory and Bridgeland stability have come out of the string theory.

My interests in algebraic geometry are broad and vast, but for the last few years I have been studying algebraic varieties by studying vector bundles on them. I often think of these as locally free "coherent sheaves" which are geometric generalizations of modules. Using the tool of Bridgeland stability, as well as a homological algebraic tool called the derived category, I have been able to address fundamental questions about vector bunddles and their moduli spaces. These are new algebraic varieties whose geometry reveals hidden geometry of the original algebraic variety, and through these moduli spaces, my work has connections with enumerative geometry (such as Gromov-Witten and Donaldson-Thomas theory) and birational geometry.

I am currently considering both more foundational and theoretical problems in the field as well as examples and applications to classical algebraic geometry. I would love to work with any students interested in doing research in any subfield of algebraic geometry.

References

[1] Igor R. Shafarevich. *Basic algebraic geometry. 1.* Springer-Verlag, Berlin, second edition, 1994. Varieties in projective space, Translated from the 1988 Russian edition and with notes by Miles Reid.