Introduction to Operator Algebras

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Credit points: 3

Summary: The theory of operator algebras is one of the richest and broadest research areas within contemporary functional analysis, having deep connections to every subject in mathematics. In fact, this topic is so huge that the research splits into several distinct branches: C*-algebras, von Neumann algebras, non-selfadjoint operator algebras, and others. Our goal in this course is to master the basics of the subject matter, get a taste of the material in every branch, and develop a high-level understanding of operator algebras.

The plan is to study the following topics:

1. Banach algebras and the basics of C*-algebras.
3. The basic theory of von Neumann algebras.
5. Introduction to operator spaces, non-selfadjoint operator algebras, and completely bounded maps.
6. Time permitting, we will learn some additional advanced topics (to be decided according to the students’ and the instructor’s interests). Possible topics:
   a. C*-algebras and von Neumann algebras associated with discrete groups.
   b. Nuclearity, tensor products and approximation techniques.
   c. Arveson’s theory of the C*-envelope and hyperrigidity.
   d. Hilbert C*-modules.

Prerequisites: I will assume that the students have taken (or are taking concurrently) the graduate course in functional analysis. Exceptional students, who are interested in this course but did not take Functional Analysis, should talk to the instructor before enrolling.

The grade: The grade will be based on written assignments, that will be presented and defended by the students.

References:

The following are good general references, though we shall not follow any of them very closely (at most a chapter here or there).

1. Orr Shalit’s lecture notes.
2. K.R. Davidson, “C*-Algebras by Example”.
4. C. Anantharaman and S. Popa, “An Introduction to II_1 Factors”.
6. V. Paulsen, “Completely Bounded Maps and Operator Algebras”.