

TECHNION - ISRAEL INSTITUTE OF TECHNOLOGY
FACULTY OF MATHEMATICS

Handout 1
General Information

Spring 2022
106929

106929 - Selected Topics in Analysis 2

Instructor: Igal Sason (office: Meyer building 652, e-mail: sason@ee.technion.ac.il).

Time and place: Monday 10:30-12:30 and Tuesday 14:30-15:30.

Pre-requisites: Algebra A, Infinitesimal Calculus 2, Introduction to Probability Theory.

Credit Points: 3.0. A joint undergraduate and graduate course.

Grading: A final homework assignment will be graded.

Course outline (tentative):

1. Basic inequalities with applications to numbers and graphs.
2. Stirling and Wallis formulas with applications.
3. Euler's constant, gamma, beta and zeta functions.
4. Applications: proving binomial identities, volumes in \mathbb{R}^n , the Dirichlet integral, integral representations of logarithmic expectations and fractional moments.
5. The arithmetic-geometric mean of Gauss, and relations to complete elliptic integrals. Geometric applications, pendulum, and a fast algorithm to compute the digits of π .
6. Euler's partial fraction expansion of the cotangent, and the Herglotz trick.
7. Application: Bernoulli numbers and the zeta function at even positive integers.
8. The Hilbert and Hardy inequalities (continuous and discrete versions).
9. Permanent of matrices, and a proof of Van der Waerden's conjecture.
10. Bregman's theorem for permanent of matrices (without proof).
11. Application: a tight asymptotic result on the number of Latin squares.
12. Selected Topics in Analytical Number Theory.
 - The infinitude of primes, and Bertrand's postulate.
 - Binomial coefficients are almost never powers.
 - Harmonic and prime-harmonic sums.

REFERENCES

- [1] M. Aigner and G. M. Ziegler, *Proofs from the Book*, Springer, 6th edition, 2018.
- [2] M. Steele, *The Cauchy-Schwarz Master Class*, Cambridge University Press, 2004.